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LETTER TO THE EDITORS

LETTER CONCERNING THE PAPER "THERMAL CONTACT RESISTANCE—THE DIRECTIONAL EFFECT AND OTHER PROBLEMS"

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IN THE appendix to their paper [1], Thomas and Probert purport to disprove my assertion [2] that thermal rectification in the contact of similar solids with different surface geometries cannot be explained as a consequence of local thermal distortion. However, there is a mathematical error in their argument which makes it impossible to accept this conclusion.

They consider the case of two, contacting, spherical asperities, one on each surface, with different radii (ρ_1, ρ_2) and derive the result

$$\left(\frac{r_{12}}{r_{21}}\right)^3 = 1 - \left\{\frac{4\Delta z(\rho_1 - \rho_2)}{b^2}\right\}^2 \tag{33}$$

(equation numbers are those of [1]), where r_{12}, r_{21} are the radius of the contact area for equal heat flow in directions $1 \to 2$, $2 \to 1$ respectively, Δz is the central normal displacement due to thermal distortion and b is the radius of the solids. They deduce that $r_{12} \neq r_{21}$ and hence that the conductance of the contact depends on the direction of heat flow.

However, the right hand side of equation (33) only differs from unity by a second order term in the small quantity Δz and, in the derivation of this result, the authors have *neglected* second order terms in $\Delta\rho_1$, $\Delta\rho_2$ which are related to Δz by the equation

$$\Delta\rho = -(2\rho^2/b^2)\,\Delta z. \tag{31}$$

If the *exact* forms of the equations are used, this residual term disappears and $r_{12} = r_{21}$.

Thus, substituting equation (30)

$$\frac{z}{b} = \frac{b}{2\rho} \tag{30}$$

into the relation (26)

$$r \propto \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)^{-\frac{1}{3}} \tag{26}$$

we get

$$r \propto \left(\frac{z_1 + z_2}{b^2}\right)^{-\frac{1}{3}}$$

and hence

$$\left(\frac{r_{12}}{r_{21}}\right)^3 = \frac{(z_1 + \Delta z_1 + z_2 - \Delta z_2)}{(z_1 - \Delta z_1 + z_2 + \Delta z_2)}$$

Thus, if $\Delta z_1 = \Delta z_2$, $(r_{12}/r_{21})^3 = 1$ and $r_{12} = r_{21}$. (A more precise analysis of this and related problems is given in [3].) The contact conductance is therefore unaffected by the thermal distortion.

This result applies to the general case of conduction of heat between two similar solids, providing that the heating conditions are such as to maintain the symmetry of the temperature field with respect to the interface. In this case, the normal displacement (Δz_1) of solid 1, due to thermal distortion, is equal and opposite to that of solid 2 (Δz_2) at all points on the surface. But, in the elastic contact of solids, only the combined profile of the surfaces ($z = z_1 + z_2$) affects the contact pressure distribution and hence the contact conductance will be unaffected by heat flow.

In the authors' experimental work with similar solids, the two solid surfaces were prepared differently and the physical properties of the surface layers would therefore probably differ. In this case, thermal distortion could cause a directional effect, but the magnitude of the observed effect makes this explanation improbable. Furthermore, the results showed no dependence on heat flow rate on which the thermal distortion produced is linearly dependent.

In all the results reported, the conductivity of the contact for upward heat flow (C_U) is greater than the corresponding downward value (C_D). This is probably coincidental, but it might result from a lack of thermal symmetry in the system. For example, if the temperature of the cooling fluid falls with depth, an additional downward heat flow through the contact would be produced, though this would be small because of the high thermal resistance between the walls of the vacuum can and the specimens. This possibility could easily be tested by carrying out an experiment with the specimen pair inverted or with two specimens with similar surfaces.

The interpretation of the negative temperature discontinuity observed with specimen pair A at low temperatures presents problems. The existence of a real heat flow against an adverse temperature gradient must surely be discounted on thermodynamic grounds, but the only other physical possibility is that the axial temperature gradient near the interface was lower than that in the body of the specimens. This could only occur if there was heat transfer from the sides of the specimens, or if the conductivity of the material near the interface was increased in some way by the contact process, since the axial heat flow was presumably uniformly distributed over the cross section away from the interface. Furthermore, such an effect would need to be considerable to account for the authors' observations. In the most extreme case reported, the overall thermal resistance of the two solids in contact (total length 5 cm) is the same as that of a single solid 1 cm less in total length. Thus, an explanation based on surface effects is not plausible.

These considerations suggest that this result must be attributed to errors in the experimental or extrapolation procedures, but the reported repeatability of the result rules out the possibility of a casual error. It is clear that the interpretation of the other directional effects observed by the authors must depend on the further investigation of this extraordinary result.

REFERENCES

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J. R. Barber

Department of Mechanical Engineering
University of Newcastle upon Tyne
NE1 7RU, England